Precalculus **Digital Learning**
Week of 4/13 **Lesson 4**

Graphs of the Other Trigonometric Functions

Print these 3 pages if you can. The first 2 pages are discussed in the video, page 3 contains a couple problems for you to try. Video – Digital Learning Lesson 4
(a link to video can be found on my website)

Learning Targets:
I can recognize the graphs of the Tangent, Cotangent, Secant and Cosecant
I can identify the vertical asymptotes on the graphs of the Tangent, Cotangent, Secant and Cosecant functions.

There are 4 other trigonometric functions that we should learn the parent graphs. The parent graphs of the Tangent, Cotangent, Secant and Cosecant are quite different from the Sine and Cosine functions. These functions can be defined as the ratios of the sine and/or cosine functions. Recall:

$\tan\theta = \frac{sin\theta}{cos\theta}$ $\cot\theta = \frac{cos\theta}{sin\theta}$ $\csc\theta = \frac{1}{sin\theta}$ and $\sec\theta = \frac{1}{cos\theta}$

As a result, each is undefined anytime the function in the denominator equals zero. On the graphs these functions we will see equally spaced vertical asymptotes. You will need to know where the asymptotes are on graphs for each of these functions. To do so we need to identify the where sine and cosine equal zero. From your unit circles or your graphs, we know:

$\sin\theta = 0\ \ when\ \theta = \ldots -180°, 0°, 180°, 360°, 540° \ldots$ or $\theta = \ldots -\pi,\ 0,\ \pi,\ 3\pi,\ 5\pi, \ldots$
$\cos\theta = 0\ \ when\ \theta = \ldots -90°, 90°, 270°, 450°, 630° \ldots$ or $\theta = \ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

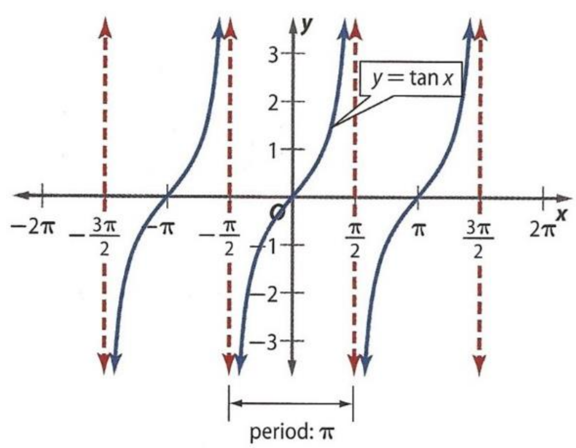
Graph of the Tangent Function

Since $\tan\theta = \frac{sin\theta}{cos\theta}$, $\tan\theta\ is\ undefined\ when\ \cos\theta = 0$

Where $\tan\theta$ *is undefined* the graph shows vertical asymptotes at:

$\theta = \ldots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

or $\theta = \frac{\pi}{2} \pm \pi n\ for\ all\ integer\ values\ of\ n$



The function is unbounded. There is **no amplitude**

The period of the function is π. $\boldsymbol{per = \frac{\pi}{b}}$

Zeroes at $\theta = 0 + \pi n\ for\ all\ integer\ values\ of\ n$

The function increases between the asymptotes.

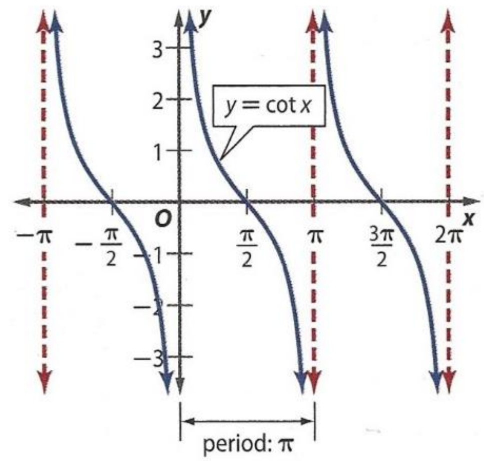
The function is Odd

$Domain = \{\theta\ /\ \theta\epsilon\mathcal{R},\ \ \theta \neq \frac{\pi}{2} \pm \pi n\ for\ all\ integer\ values\ of\ n\}$

$Range = \{y\ /\ y\epsilon\mathcal{R}\}$

Graph of the Cotangent Function

Since $\cot\theta = \frac{cos\theta}{sin\theta}$, $\cot\theta$ *is undefined when* $\sin\theta = 0$,



Vertical asymptotes wher $\sin\theta = 0$:

$\theta = \;. . . -\pi, 0, \pi, 2\pi, . . .$

or $\theta = 0 \pm \pi n$ *for all integer values of n*

The function is unbounded. There is **<u>no amplitude</u>**

The period of the function is π. $\boldsymbol{per = \frac{\pi}{b}}$

Zeroes at $\theta = \frac{\pi}{2} + \pi n$ *for all integer values of n*

The function decreases between the asymptotes.

The function is Odd

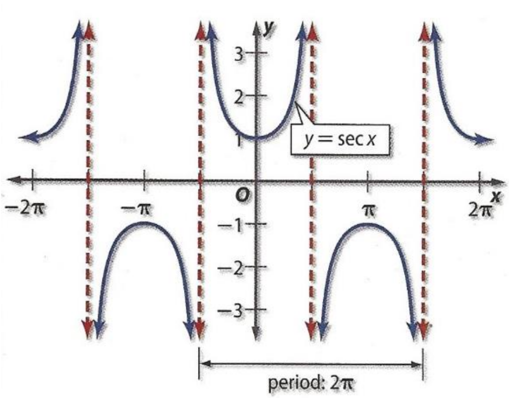
$Domain = \{\theta \,/\, \theta \epsilon \boldsymbol{\mathcal{R}},\;\; \theta \neq 0 \pm \pi n \text{ for all integer values of } n\}$

$Range = \{y \,/\; y \epsilon \boldsymbol{\mathcal{R}}\}$

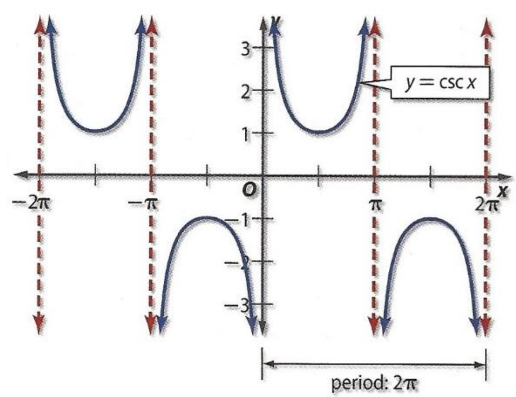
Graph of the Secant and Cosecant Functions

Since $\sec\theta = \frac{1}{cos\theta}$ and $\csc\theta = \frac{1}{sin\theta}$, they too will have vertical asymptotes.

$y = \sec x$



$y = \csc x$



Vertical asymptotes at x = $. . -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, . .$

or $x = \frac{\pi}{2} \pm \pi n$

Vertical asymptotes $x = \;. . -\pi, 0, \pi, 2\pi, . .$

or $x = 0 \pm \pi n$

Both functions are unbounded. There are **<u>no amplitudes</u>**

The period of both functions is 2π. $\boldsymbol{per = \frac{2\pi}{b}}$

Neither functions have zeroes.

$Domain = \{x \,/\, x \epsilon \boldsymbol{\mathcal{R}},\;\; x \neq \frac{\pi}{2} \pm \pi n\}$

$Range = \{y \,/\; y \leq -1 \;\; \cup \;\; y \geq \; 1\}$

$Domain = \{x \,/\, x \epsilon \boldsymbol{\mathcal{R}},\;\; x \neq 0 \pm \pi n\}$

$Range = \{y \,/\; y \leq -1 \;\; \cup \;\; y \geq \; 1\}$

Pause the video for a few minutes and try questions these 3 multiple choice questions, then resume to check your results.

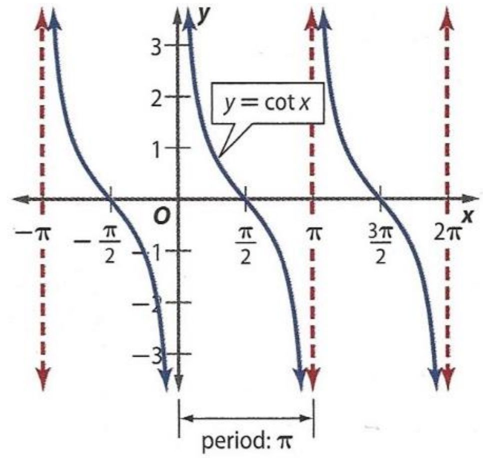
1) Which statement correctly identifies the vertical asymptotes of $y = \cot x$

a) $x = \frac{\pi}{2} \pm \pi n$ for all integer values of n

b) $x = 0 \pm \pi n$ for all integer values of n

c) $x = \frac{\pi}{2} \pm \frac{\pi}{2} n$ for all integer values of n

d) $x = 0 \pm \frac{\pi}{2} n$ for all integer values of n



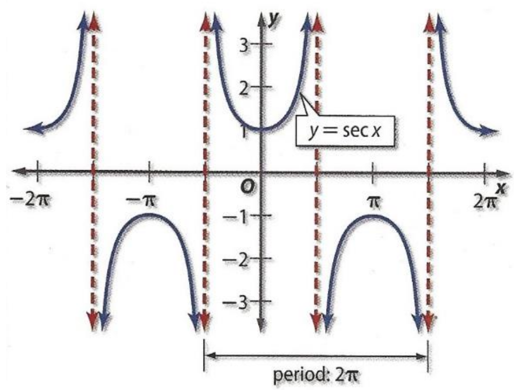
2) Which statement correctly identifies the vertical asymptotes of $y = \sec x$

a) $x = \frac{\pi}{2} \pm \frac{\pi}{2} n$ for all integer values of n

b) $x = 0 \pm \frac{\pi}{2} n$ for all integer values of n

c) $x = \frac{\pi}{2} \pm \pi n$ for all integer values of n

d) $x = 0 \pm \pi n$ for all integer values of n



3) Which of the following pairs of statements are true for $y = \tan x$ for all integer values of n

a) $Domain = \{x \,/\, x \epsilon \mathcal{R},\ \ x \neq \frac{\pi}{2} \pm \pi n\}$
$Range = \{y \,/\ \ y \leq -1 \ \ \cup \ \ y \geq 1\}$

b) $Domain = \{x \,/\, x \epsilon \mathcal{R},\ \ \theta \neq 0 \pm \pi n\}$
$Range = \{y \,/\ \ y \epsilon \mathcal{R}\}$

c) $Domain = \{x \,/\, x \epsilon \mathcal{R},\ \ x \neq 0 \pm \pi n\}$
$Range = \{y \,/\ \ y \leq -1 \ \ \cup \ \ y \geq 1\}$

d) $Domain = \{x \,/\, x \epsilon \mathcal{R},\ \ \theta \neq \frac{\pi}{2} \pm \pi n\}$
$Range = \{y \,/\ \ y \epsilon \mathcal{R}\}$